1 **METHOD AND APPARATUS FOR SELF SERVOWRITING OF TRACKS OF A DISK**

2 **DRIVE USING A SERVO CONTROL LOOP HAVING A TWO-DIMENSIONAL**

3 **WEIGHTED DIGITAL STATE COMPENSATOR**

4 This application is a continuation-in-part of Application No. 10/280,603, filed October 24,

5 2003, herein incorporated by reference

6 **BACKGROUND OF THE INVENTION**

7 **Field of the Invention**

8 The present invention relates to magnetic disk drives, and more particularly, to self

9 servowriting of tracks on a rotating magnetic disk medium.

10 **Description of the Prior Art**

11 The writing of servotrack information on a magnetic disk medium is a critical process in

12 the manufacture of a disk drive. Conventionally, servotrack information is written with a

13 specialized servowriting instrument mounted on a large granite block to minimize external

14 vibration effects. Increasing track densities and decreasing disk-drive size has led to the

15 investigation of self servowriting techniques. One issue confronting the use of self servowriting is

16 track-to-track or radial error propagation and amplification of written-in errors and imperfections

17 with respect to a perfectly circular track.

18 U.S. Patent No. 5,907,447 to Yarmchuk et al. describes reduction of radial error

19 propagation by generating a correction signal using a filter applied to a position error signal (PES)

20 to reduce a closed-loop response of a track-following servo loop to less than unity at frequencies

21 equal to integer multiples of the disk rotation frequency. While permitting implementation of self

22 servowriting with reduced radial error propagation, the PES filtering technique of the Yarmchuk

23 patent fails to readily support increasingly aggressive track densities.

24 Accordingly, there exists a need for technique for aggressively reducing written-in error

25 propagation during self servowriting. The present invention satisfies these needs.

26

27 **SUMMARY OF THE INVENTION**

28 The present invention may be embodied in a method, implemented in a magnetic disk

1   drive, for writing servo burst patterns for tracks on a rotating magnetic disk medium. In the

2   method, a first reference track, defined by previously written servo burst patterns, is followed

3   using a servo control loop while writing servo burst patterns at a first target radial location on the

4   magnetic disk medium. The servo control loop has a closed-loop response and includes a two-

5   dimensional digital state compensator having first and second inputs and first and second outputs.

6   The first input receives position error signals and the first output generates control signals for

7   positioning a transducer head with respect to the selected track during track following. The

8   second output generates track-following state variables during track following, and the second

9   input receives combined track-following state variables. Accordingly, the track-following state

10   variables generated at the second output during the writing of the servo burst patterns on the first

11   target radial location are processed and stored. A second reference track, defined by previously

12   written servo burst patterns, is followed using the servo control loop while writing servo burst

13   patterns at a second target radial location. The track-following state variables generated at the

14   second output during the writing of the servo burst patterns at the second target radial location

15   are processed and stored. A third reference track, defined by the previously written servo burst

16   patterns at the first and second radial target locations, is followed using the servo control loop

17   while writing servo burst patterns at a third target radial location. The processed and stored

18   track-following state variables generated at the second output during writing of the servo burst

19   patterns at the first and second target radial locations are combined, and the combined track

20   following state variables are applied to the second input during writing of the servo burst patterns

21   at the third target radial location.

22   In more detailed features of the invention, the dimensions of the two-dimensional digital

23   state compensator may be circumferential position and radial position. The first reference track

24   may be offset from the second reference track by more than one servo track, and the third

25   reference track may be offset from the second reference track by at least one servo track. Each

26   radial location may be offset from the corresponding reference track by at least one servo track.

27   Also, processing of the track-following state variables generated at the second output may include

28   weighting and time shifting the track-following state variables generated at the second output.

29   Further, the compensator may be defined by the following equations 1-3:

1
$$\begin{bmatrix} \hat{x}_k(t+1) \\ \tilde{y}_{k+1}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} e_k(t) \qquad (1)$$

2
$$u_k(t) = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \qquad (2)$$

3
$$\hat{y}_{k+1}(t) = \sum_{j=1}^{T} \alpha_j \tilde{y}_{k+1}(j) \qquad (3)$$

4   where

5   k is the reference track number;

6   t is a servo wedge number or time;

7   $u_k(t)$ is the control signals for positioning the transducer head,

8   $x_k(t)$ is a state vector in a first dimension or time,

9   $\hat{y}_k(t)$ is the combined track-following state variables, in a second dimension or track

10   number,

11   $\tilde{y}_{k+1}(t)$ is the track-following state variables generated during writing of the servo burst

12   patterns,

13   $\hat{y}_{k+1}(t)$ is the weighted, time-shifted track-following state variable obtained from $\tilde{y}_{k+1}(t)$,

14   T is the total number of servo wedges per track,

15   α_j are weight values,

16   $e_k(t)$ is the position error signals (PES), and

17   $A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}, C_{11}, C_{12}$ are matrices of appropriate dimensions.

18   Alternatively, the compensator may be defined (in observer based form) by the following

19   equations 4-7:

20
$$\begin{bmatrix} \hat{x}_k(t+1) \\ \tilde{y}_{k+1}(t) \end{bmatrix} = \begin{bmatrix} A_g & 0 \\ C_g & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} B_g \\ 0 \end{bmatrix} u_k(t) + K_e \left(e_k(t) - \hat{e}_k(t) \right) \qquad (4)$$

21
$$\hat{e}_k(t) = \begin{bmatrix} -C_g & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \qquad (5)$$

22
$$u_k(t) = K_c \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \qquad (6)$$

1
$$\hat{y}_{k+1}(t) = \sum_{j=1}^{T} \alpha_j \tilde{y}_{k+1}(j)$$
(7)

2   where

3   k is the reference track number;

4   t is a servo wedge number or time;

5       $e_k(t)$ is the position error signals (PES),

6       $\hat{e}_k(t)$ is an estimate of the PES,

7   (A_g, B_g, C_g) is a state space description of a head disk assembly,

8       $u_k(t)$ is the control signals for positioning the transducer head

9       $x_k(t)$ is a state vector in the first dimension or time,

10      $\hat{y}_k(t)$ is the combined track-following state variable, in a second dimension or track

11 number,

12      $\tilde{y}_{k+1}(t)$ is the track-following state variables generated during writing of the servo burst

13 patterns,

14      $\hat{y}_{k+1}(t)$ is the weighted, time-shifted track-following state variable obtained from $\tilde{y}_{k+1}(t)$,

15   T is the total number of servo wedges per track,

16   α_j are weight values,

17   K_e is an estimator gain, and

18   K_c is a compensator gain.

19   In an alternative embodiment of the invention, a reference track, defined by previously

20 formed servo burst patterns on the magnetic disk medium, is followed using a servo control loop

21 while forming servo burst patterns defining a first target track. The servo loop has a closed-loop

22 response and includes a two-dimensional digital state compensator having first and second inputs

23 and first and second outputs. The first input receives position error signals (PES) and the first

24 output generates control signals for positioning a transducer head with respect to the selected

25 track during track following. The second output generates track-following state variables during

26 track following, and the second input receives processed and stored track-following state

27 variables. Accordingly, the track-following state variables generated at the second output while

1 forming the servo burst patterns defining the first target track are processed and stored. The first

2 target track is followed using the servo control loop while forming servo burst patterns defining a

3 second target track. While forming the servo burst patterns defining the second target track, the

4 processed and stored track-following state variables generated at the second output while forming

5 the servo burst patterns defining the first target track are applied to the second input.

6   In more detailed features of the invention, the dimensions of the two-dimensional digital

7 state compensator may be circumferential position and radial position. The first target track may

8 be offset from the reference track by one servo track, and the second target track may be offset

9 from the first target track by one servo track. Alternatively, the first target track may be offset

10 from the reference track by more than one servo track, and the second target track may be offset

11 from the first target track by more than one servo track.

12   Further, the compensator may be defined by the following equations 8-10:

13
$$\begin{bmatrix} \hat{x}_k(t+1) \\ \tilde{y}_{k+1}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} e_k(t) \tag{8}$$

14
$$u_k(t) = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \tag{9}$$

15
$$\hat{y}_{k+1}(t) = \sum_{j=1}^{T} \alpha_j \tilde{y}_{k+1}(j) \tag{10}$$

16 where

17   k is the track number;

18   t is a servo wedge number or time;

19     $u_k(t)$ is the control signals for positioning the transducer head,

20     $x_k(t)$ is a state vector in a first dimension or time,

21     $\hat{y}_k(t)$ is the stored track-following state variables, in a second dimension or track number,

22     $\tilde{y}_{k+1}(t)$ is the track-following state variables that are stored while forming the servo burst

23 patterns,

24     $\hat{y}_{k+1}(t)$ is the weighted, time-shifted track-following state variable obtained from $\tilde{y}_{k+1}(t)$,

25   T is the total number of servo wedges per track,

26   α_j are weight values,

1          $e_k(t)$ is the position error signals (PES), and

2          $A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}, C_{11}, C_{12}$ are matrices of appropriate dimensions.

3   Alternatively, the compensator may be defined (in observer-based form) by the following

4 equations:

5
$$\begin{bmatrix} \hat{x}_k(t+1) \\ \tilde{y}_{k+1}(t) \end{bmatrix} = \begin{bmatrix} A_g & 0 \\ C_g & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} B_g \\ 0 \end{bmatrix} u_k(t) + K_e \left(e_k(t) - \hat{e}_k(t) \right) \tag{11}$$

6
$$\hat{e}_k(t) = \begin{bmatrix} -C_g & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \tag{12}$$

7
$$u_k(t) = K_c \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \tag{13}$$

8
$$\hat{y}_{k+1}(t) = \sum_{j=1}^{T} \alpha_j \tilde{y}_{k+1}(j) \tag{14}$$

9 where

10   k is the track number;

11   t is a servo wedge number or time;

12          $e_k(t)$ is the position error signals (PES),

13          $\hat{e}_k(t)$ is an estimate of the PES,

14   (A_g, B_g, C_g) is a state space description of a head disk assembly,

15          $u_k(t)$ is the control signals for positioning the transducer head

16          $x_k(t)$ is a state vector in the first dimension or time,

17          $\hat{y}_k(t)$ is the track-following state variable, in a second dimension or track number,

18          $\tilde{y}_{k+1}(t)$ is the track-following state variables that are processed and stored while forming

19 the servo burst patterns,

20          $\hat{y}_{k+1}(t)$ is the weighted, time-shifted track-following state variable obtained from $\tilde{y}_{k+1}(t)$,

21   T is the total number of servo wedges per track,

22   α_j are weight values,

23   K_e is an estimator gain, and

1   K_c is a compensator gain.

2

3   **BRIEF DESCRIPTION OF THE DRAWINGS**

4   The accompanying drawings illustrate embodiments of the present invention and, together

5   with the description, serve to explain the principles of the invention.

6   Figure 1 is a flow chart illustrating a first embodiment of a self servo-writing method for

7   defining tracks on a rotating magnetic disk medium of a disk drive, using a track-following servo-

8   control loop having a two-dimensional digital state compensator, according to the present

9   invention.

10   Figure 2 is a block diagram of the track-following servo control loop.

11   Figure 3 is a block diagram of a disk drive coupled to a host for implementing the

12   determining method of Figure 1.

13   Figure 4 is a schematic diagram illustrating a transducer head, having non-overlapping

14   read and write elements offset by at least two servo tracks, for self-servo writing write-element-

15   width servo bursts for defining servo tracks, according to the first embodiment of the present

16   invention.

17   Figure 5A is a schematic diagram illustrating ideal servo tracks on a disk of a disk drive.

18   Figure 5B is a schematic diagram illustrating written servo tracks exhibiting imperfections.

19   Figure 6 is a flow chart illustrating a second embodiment of a self servo-writing method

20   for defining tracks on a rotating magnetic disk medium of a disk drive, using a track-following

21   servo-control loop having a two-dimensional digital state compensator, according to the present

22   invention.

23   Figure 7 is a schematic diagram illustrating a transducer head, having non-overlapping

24   read and write elements offset by at least two servo tracks, for self-servo writing servo bursts for

25   defining servo tracks, according to the second embodiment of the present invention.

26

1   **DETAILED DESCRIPTION**

2   With reference to Figures 1 through 4, the present invention may be embodied in a method

3   100 (Figure 1), implemented in a magnetic disk drive 30 (Figure 3), for writing servo burst

4   patterns, A, B, C and D (Figure 4), for servo tracks 12 on a rotating magnetic disk medium. In

5   the method, a first reference track N+1, defined by previously written servo burst patterns, C and

6   D, is followed using a servo control loop 24 (Figure 2) while writing servo burst patterns A at a

7   first target radial location coincident with yet to be defined servo track N+3 (step 112). The

8   servo loop has a closed-loop response and includes a two-dimensional digital state compensator

9   26 having first and second inputs, I_1 and I_2, and first and second outputs, O_1 and O_2. The first

10   input receives position error signals (PES) and the first output generates control signals $u_k(t)$ for

11   positioning a transducer head 42 with respect to the selected track during track following. The

12   second output generates track-following state variables $\tilde{y}_{k+1}(t)$ during track following, and the

13   second input receives processed and combined track-following state variables $\hat{y}_k(t)$.

14   Accordingly, the track-following state variables generated at the second output during the writing

15   of the servo burst patterns on the first target radial location are processed by element 27 discussed

16   below and stored (step 114) in, for example, a table 28. A second reference track N+3, defined

17   by previously written servo burst patterns, C and D, is followed using the servo control loop while

18   writing servo burst patterns B at a second target radial location coincident with the yet to be

19   defined servo track N+5 (step 116). The track-following state variable generated at the second

20   output during writing of the servo burst patterns at the second target radial location are processed

21   and stored (step 118). A third reference track N+4, defined by the previously written servo burst

22   patterns, A and B, at the first and second radial target locations, is followed using the servo

23   control loop while writing servo burst patterns D at a third target radial location coincident with

24   the yet to be defined servo track N+6 (step 120). The processed and stored track-following state

25   variables generated at the second output during writing of the servo burst patterns at the first and

26   second target radial locations are combined, and the combined track following state variables are

27   applied to the second input during writing of the servo burst patterns at the third target radial

28   location.

29   Advantageously, seed servo burst patterns, A, B, C and D, for initially defining servo

30   tracks 12, N and N+1, are written using a technique for forming as near perfect circular track path

1   as practical. Also, interleaved permanent and temporary servo burst patterns may be used to

2   address timing and sensor element cross-talk issues. Accordingly, the permanent servo burst

3   patterns may be used for track following while the temporary servo burst patterns are written, and

4   the temporary servo burst patterns may be used for track following while the permanent servo

5   burst patterns are written. After servo tracks are written across the entire disk surface, the

6   temporary servo burst patterns would then revert to data sectors of the corresponding data tracks

7   and eventually would be overwritten by user data. Also, while not explicitly described above, the

8   servo burst patterns C centered on servo track N+4 may be written as a seed track or may be

9   written by track following along, for example, servo track N+2.

10   When writing servo burst patterns at a target radial location, the PES from a reference

11   track 12 is created by reading at least two servo burst pattern edges from, for example, patterns A

12   and B that define the track N+4. Each of these patterns has associated with it a retrieved stored

13   state variable $\hat{y}_k$, say $\hat{y}_k^A$ and $\hat{y}_k^B$. The effective stored state variables to be inputted to the

14   second input I_2 of the two-dimensional compensator when writing servo burst patterns at the

15   target radial location is a linear combination of the two stored state variables, i.e.,

16    $\hat{y}_k = f\left(\hat{y}_k^A, \hat{y}_k^B\right)$. The state output variable at the second output O_2 of the compensator will then

17   be processed and stored with reference to the presently-written servo burst pattern at the target

18   radial location. The processing of the second state output variable O_2 may include weighting and

19   time-shifting the state variable, represented by element 27. The coefficients of the linear

20   combination of stored state variables may be proportional to the magnitudes of the servo burst

21   pattern elements, in this example A and B, although modifications to this rule may be made to

22   accommodate or correct for such imperfections such as PES linearity of the read element.

23   The dimensions of the two-dimensional digital state compensator 26 may be

24   circumferential position or time t, and radial position or track number k. The first reference track

25   N+1 may be offset from the second reference track N+3 by more than one servo track, and the

26   third reference track N+4 may be offset from the second reference track by at least one servo

27   track. Each radial location may be offset from the corresponding reference track by at least one

28   servo track.

29   The disk drive 30 (Figure 3) has a head disk assembly (HDA) 32 and a sampled servo

1   controller 34. The HDA includes a rotating magnetic disk 36 having, after servo writing,

2   distributed position information in a plurality of uniformly spaced-apart servo wedges 38, a rotary

3   actuator 40 that pivots relative to a base and that carries a transducer head 42 that periodically

4   reads the position information from the servo wedges, and a voice coil motor (VCM) circuit 44

5   that includes a voice coil motor coupled to the rotary actuator and that responds to a control

6   effort signal 46. The sampled servo controller periodically adjusts the control effort signal during

7   a track-following operation based on the position information. Advantageously, the transducer

8   head has a read element 48 and an offset write element 50 (Figure 4). Generally, the write

9   element is wider than the read element. The center of the read element may be spaced from a

10   nearest edge of the write element by at least one servo track such that the elements do not

11   overlap.

12   An ideal servo track 12 is one that forms a perfect circle on the disk 36, as shown in

13   Figure 5A. During manufacture, servo information for the embedded servo wedges 38 is placed

14   on the disk during the self servowriting operation. The servo information includes the servo burst

15   patterns having edges that are placed at locations that deviate outwardly or inwardly from the

16   ideal "center line" of the servo track circle as shown in Figure 5B. These apparent deviations

17   from the ideal servo track center line can occur due to spindle runout, vibrations or movements

18   during servo writing operation, and media defects or noise in the region of the servo burst

19   patterns.

20   The disk drive 30 further includes a control system 54, and the HDA 32 further includes a

21   spindle motor 60 and a preamplifier 66. The control system includes the sampled servo controller

22   34, and circuitry and processors that control the HDA and that provide an intelligent interface

23   between a host 58 and the HDA for execution of read and write commands. The control system

24   may have an internal microprocessor and nonvolatile memory for implementing the techniques of

25   the invention. Program code for implementing the techniques of the invention may be stored in

26   the nonvolatile memory and transferred to volatile random access memory (RAM) for execution

27   by the microprocessor.

28   The data tracks on the media surface may be divided into the storage segments. Each

29   storage segment may begin with a sector of the servo wedges 38 which is followed by data

30   sectors. The data sectors may include data blocks, each generally storing 512 data bytes. Each

1   data block may be addressed using a logical block address (LBA).

2   The servo control loop 24 (Figure 2) includes the HDA 32 after the track-following

3   compensator 26. Disturbances ς_k to the HDA alter the resulting head position y_{k+1}. A track

4   selection signal y_k is compared to the head position y_{k+1} to generate the PES. When a track 12 is

5   defined, the data (one number per servo wedge) from the second output O_2 of the compensator

6   26 is stored in the lookup table 28 and used as the second input I_2 to the compensator when the

7   next track is defined. This process is repeated for defining all of the tracks during the self

8   servowriting process. A state-space model for the HDA 32 is given by

9    $$x_k^g(t+1) = A_g x_k^g(t) + B_g u_k(t) \tag{15}$$

10   $$y_{k+1}(t) = C_g x_k^g(t) + \varsigma_k(t) \tag{16}$$

11   with ς_k being the output disturbance. The PES output is given by

12   $$e_k(t) = \begin{bmatrix} -C_g & 1 \end{bmatrix} \begin{bmatrix} x_k^g \\ y_k \end{bmatrix} + \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \varsigma_k \\ \omega_k \end{bmatrix} \tag{17}$$

13   where ω_k is the head sensor noise.

14   A two-dimensional observer-based compensator 26 for the system is given by

15   $$\begin{bmatrix} \hat{x}_k(t+1) \\ \tilde{y}_{k+1}(t) \end{bmatrix} = \begin{bmatrix} A_g & 0 \\ C_g & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} B_g \\ 0 \end{bmatrix} u_k(t) + K_e \left(e_k(t) - \hat{e}_k(t) \right) \tag{18}$$

16   $$\hat{e}_k(t) = \begin{bmatrix} -C_g & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \tag{19}$$

17   with

18   $$u_k(t) = K_c \begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \tag{20}$$

19   where K_c is the compensator gain, and K_e is an estimator gain. Let

20   $$K_e = \begin{bmatrix} K_e^1 \\ K_e^2 \end{bmatrix}; \quad K_c = \begin{bmatrix} K_c^1 & K_c^2 \end{bmatrix} \tag{21}$$

21   Substituting for $\hat{e}_k(t)$ and $u_k(t)$ gives the observer-based controller in output feedback form:

1

$$\begin{bmatrix} \hat{x}_k(t+1) \\ \tilde{y}_{k+1}(t) \end{bmatrix} = \begin{bmatrix} A_g & 0 \\ C_g & 0 \end{bmatrix}\begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} B_g \\ 0 \end{bmatrix}\begin{bmatrix} K_c^1 & K_c^2 \end{bmatrix}\begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix}$$

$$+ K_e e_k(t) - \begin{bmatrix} K_e^1 \\ K_e^2 \end{bmatrix}\begin{bmatrix} -C_g & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} \tag{22}$$

$$= \begin{bmatrix} A_g + B_g K_c^1 + K_e^1 C_g & B_g K_c^2 - K_e^1 \\ C_g + K_e^2 C_g & -K_e^2 \end{bmatrix}\begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix} + \begin{bmatrix} K_e^1 \\ K_e^2 \end{bmatrix}e_k(t)$$

2

$$u_k(t) = \begin{bmatrix} K_c^1 & K_c^2 \end{bmatrix}\begin{bmatrix} \hat{x}_k(t) \\ \hat{y}_k(t) \end{bmatrix}, \tag{23}$$

3

$$\hat{y}_{k+1}(t) = \sum_{j=1}^{T} \alpha_j \tilde{y}_{k+1}(j) \tag{24}$$

4   Equations 22-23 are of the same form as equations 1-2, and the matrices

5   $A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{21}, C_{11}, C_{12}$ of equations 1 and 2 may be obtained by comparison.

6   The closed-loop system is formed by grouping the "x" and "y" states of the HDA 32 and

7   compensator 26 together. Denoting

8

$$\overline{x}_k = \begin{bmatrix} x_k^g \\ \hat{x}_k \end{bmatrix}, \tag{25}$$

9

$$\overline{y}_k = \begin{bmatrix} y_k \\ \tilde{y}_k \end{bmatrix} \tag{26}$$

10   the closed loop system is given by

11

$$\begin{bmatrix} \overline{x}_k(t+1) \\ \overline{y}_{k+1}(t) \end{bmatrix} = \overline{A}\begin{bmatrix} \overline{x}_k(t) \\ \overline{y}_k(t) \end{bmatrix} + \overline{B}w_k(t) \tag{27}$$

12

$$\hat{y}_{k+1}(t) = \sum_{j=1}^{T} \alpha_j \tilde{y}_{k+1}(j) \tag{28}$$

13   where

14

$$\overline{A} = \begin{bmatrix} A_g & B_g K_c^1 & 0 & B_g K_c^2 \\ -K_e^1 C_g & A_g + B_g K_c^1 + K_e^1 C_g & K_e^1 & B_g K_c^2 - K_e^1 \\ C_g & 0 & 0 & 0 \\ -K_e^2 C_g & C_g + K_e^2 C_g & K_e^2 & -K_e^2 \end{bmatrix}, \tag{29}$$

1
$$\overline{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$
(30)

2
$$\overline{w}_k(t) = \begin{bmatrix} \varsigma_k(t) \\ \omega_k(t) \end{bmatrix},$$
(31)

3 $\tilde{y}_{k+1}(t)$ is the track-following state variables generated during writing of the servo burst

4 patterns,

5 $\hat{y}_{k+1}(t)$ is a weighted, time-shifted track-following state variable obtained from $\tilde{y}_{k+1}(t)$,

6 T is the total number of servo wedges per track, and

7 α_j are weight values.

8 With reference to Figures 6 and 7, the present invention may be embodied in a self servo-

9 writing method 160 (Figure 6), implemented in the magnetic disk drive 30 (Figure 3), for defining

10 servo tracks 12 (Figure 7) on a rotating magnetic disk medium. In the method, a reference track

11 N, defined by previously formed servo burst patterns, A and B, is track followed using a servo

12 control loop 24 (Figure 2) while forming servo burst patterns, C and D, defining a first target

13 track N+1 (step 162). The servo control loop has a closed-loop response and includes a two-

14 dimensional digital state compensator 26 having first and second inputs, I_1 and I_2, and first and

15 second outputs, O_1 and O_2. The first input receives position error signals (PES) and the first

16 output generates control signals $u_k(t)$ for positioning a transducer head 42 with respect to the

17 selected track during track following. The second output generates track-following state

18 variables $\tilde{y}_{k+1}(t)$ during track following, and the second input receives processed and stored

19 track-following state variables $\hat{y}_k(t)$. Accordingly, the track-following state variables generated

20 at the second output while forming the servo burst patterns defining the first target track are

21 processed and stored (step 164). The first target track is track followed using the servo control

22 loop while servo burst patterns, A' and B, are formed defining a second target track N+2. While

23 forming the servo burst patterns defining the second target track, the processed and stored track-

24 following state variables generated at the second output while forming the servo burst pattern

25 defining the first target track are applied to the second input (step 166).

1   The first target track N+1 may be offset from the reference track N by one servo track,

2   and the second target track N+2 may be offset from the first target track by one servo track.

3   Alternatively, the tracks may be offset by more than one servo track.

4   Self servowriting techniques for forming the servo burst patterns to define the servo tracks

5   is described in more detail in U.S. patent application serial number 09/541,136, filed on March 31,

6   2000, and titled SELF-SERVO WRITING A DISK DRIVE BY PROPAGATING

7   INTERLEAVED SETS OF TIMING CLOCKS AND SERVO BURSTS DURING

8   ALTERNATE TIME INTERVALS, which is incorporated herein by reference.